August 2003: Use At Least Twelve Observations in Constructing a Confidence Interval (Rule 1.10)

Introduction

Rules of the month are numbered in accordance with the numbering in the book. Thus, Rule 1.1 refers to the first rule in Chapter 1. And so on. These comments do not repeat the material in the book but highlights and amplifies it. A rule is stated as found in the book and then discussed.

Rule 1.10 "The width of the confidence interval, involving estimation of variability and sample size, decreases rapidly until 12 observations are reached and then decreases less rapidly."

Further Comments on the Rule

A paper by Parker and Berman (2003) leads to the further consideration that the basic formula is the beginning for sample size calculations but not the end. These authors discuss the topic that sample size involves more than calculations. Specifically, they argue that the amount of information associated with a particular sample size may be more useful than linking the sample size to a difference to be detected. Their point is well-taken. The challenge is to quantify the information in some useful way. One approach they take is to consider the width or half-width of a confidence interval as a function of sample size. To make this work in general it is necessary that observations are expressed in units of the standard deviation. In specific instances it may be possible to use a standard deviation based on, say, previous work so that the width of the confidence interval can be related directly to the units of the observations.

This approach is also discussed in Rule 1.10 above. It suggests that the width of the confidence decreases rapidly until about 12 observations and then tapers off—although always decreasing—see Figure 1.2 in the book.

A further consideration is the following. The half-width of the confidence interval, call it w, for n observations and assuming the observations have been standardized to have standard deviation 1, is

$$w = \frac{t_{n-1,1-\alpha/2}}{\sqrt{n}}.$$

The amount of information in such an interval is proportional to the square of the reciprocal of w.

$$I(n,\alpha) = \frac{n}{t^2_{n-1,1-\alpha/2}}. \qquad (1)$$

This is a somewhat non-standard definition of information but has the advantage that it explicitly incorporates the information when the variance is unknown. If the variance were known, then $I(n,\alpha)$ would be simply proportional to n.

The incremental amount of information, as defined above, per unit increase in sample size is remarkably constant. It is given by,

$$\Delta I(n,\alpha) = I(n,\alpha) - I(n-1,\alpha) = \frac{n}{t^2_{n-1,1-\alpha/2}} - \frac{n-1}{t^2_{n-2,1-\alpha/2}}..$$

A graph of D(n,α) against n for a values of 0.90, 0.95, 0.99 is given in Figure 1.

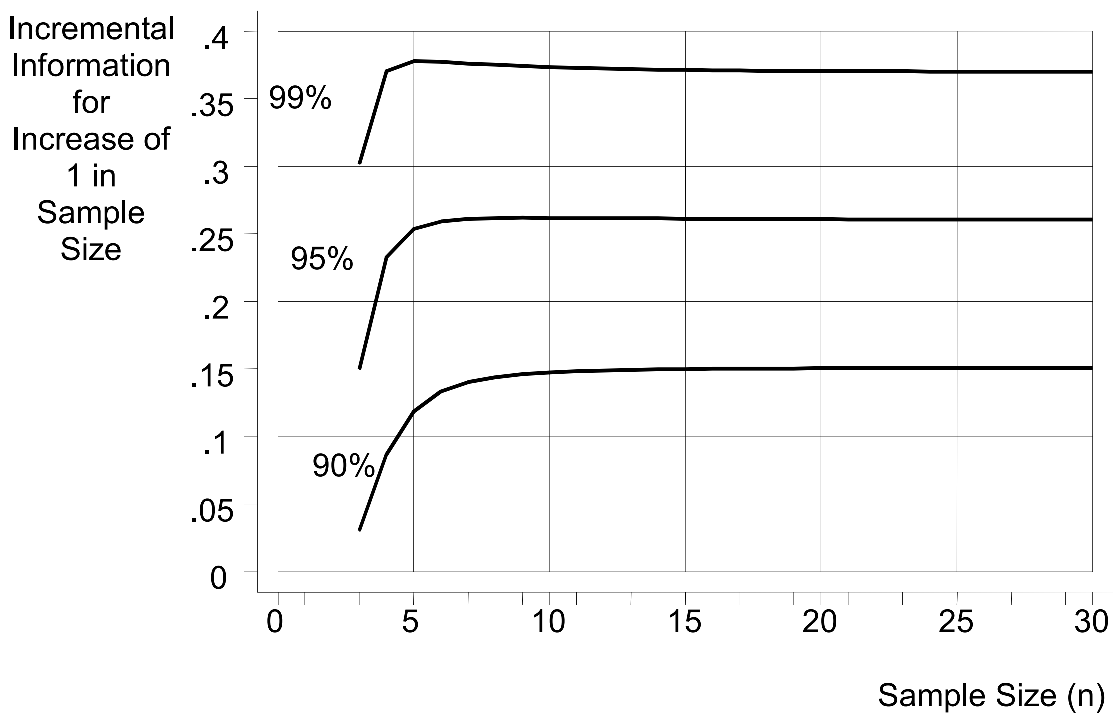

Figure 1. Incremental information in a single observation as related to the sample size. Beyond 10 the incremental information is virtually constant and asymptotically equal to $(1/z_{1-\alpha/2})^2$ where z is a standard normal deviate.

The incremental information can approximated by $(1/z_{1-\alpha/2})^2$ since $z_{1-\alpha/2}$ is the asymptotic value of the *t*-statistic. Equations (1) and (2) could have been standardized by dividing the *t*-statistic by $z_{1-\alpha/2}$. This would shift the curves but the pattern will remain the same.

The implicit rationale for the rule is that with ten or twelve observations the variance is becoming known with reasonable precision and the focus can be shifted to the issue of location effects.

Parker and Berman (2003) provide three interesting examples illustrating sample size considerations from the point of view of the amount of information in the sample.

Reference

Parker, R.A. and Berman, N.G. (2003). Sample size: more than calculations. *The*

American Statistician, **57:** 166-170.

Acknowledgement

I very much appreciate a discussion with Roger Higdon, National Alzheimer's Coordinating Center, University of Washington about this month's topic.